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Fock Theories and Quantum Logics

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The notion of Fock theory is introduced in the framework of quantum logics, which are here orthomodular atomic lattices satisfying the covering property. It is shown that there are some fundamental facts concerning particles, which may be successfully discussed in this general context. One of these facts is to establish the theoretical conditions for considering particles as sharply defined entities. The other refers to the theoretical circumstances, which almost impose to consider that some particles have a structure, meaning they are composed from other particles. This last problem is strongly related with the conservative time evolutions.

KEY WORDS: fock theories; orthomodular lattices; many particles; time evolutions; quantum logics.

1. INTRODUCTION

In what follows we will prefer to use the term of physical theory instead of quantum logic. In order to avoid any confusion we give below the list of principal mathematical terms used in our text. For physical meaning of some terms or physical interpretations of results italics will be used.

By a physical theory, $(L, \leq, \perp)$ where L is a nonempty set, $\leq$ is a partial order relation, and $\perp$ is an orthocomplementation on L is meant an orthomodular atomic lattice having the covering property. Remember that a lattice L is atomic if under any element of L there is an atom, i.e. a minimal element of L. The set of all atoms of L will be denoted by $\Omega(L)$. The lattice L satisfies the covering property if, given $a \in L$, $p \in \Omega(L)$, there is no element between a and $p \vee a$. The notations $a \vee b$ and $a \wedge b$ denote the lowest upper bound (the join) and the greatest lower bound (the meet) of a and b, respectively. We say that the elements $a, b \in L$ commute/are compatible if the following equality holds: $a = (a \wedge b) \vee (a \wedge b^{\perp})$, where $b^{\perp}$ denotes the orthocomplement of b. In this situation we write $(a, b)C$. If two elements $a, b \in L$ have the property $a \leq b^{\perp}$, then we say that they are orthogonal and write $a \perp b$ or $(a, b)\perp$. An orthomodular lattice having the property that all its elements commute is called a Boolean algebra. *From the physical point*

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of view the elements of a physical theory represent "yes-no" experiments or simply tests. When two elements of L commute we say that their corresponding tests are empirically compatible, which means that there exists an experimental procedure permitting to measure them simultaneously.

Any subset of L which is itself a lattice with the order and orthocomplementation inherited from L, is called an orthosublattice of L. It is easy to prove that any physical theory may be represented as the union of a family of its orthosublattices, which are atomic Boolean algebras. It is interesting to mention that physical theories may have orthosublattices that are Boolean algebras having no atoms. Now we can define the notion of state of a physical theory. A state on L is a mapping from L to [0, 1], which, restricted to any Boolean orthosublattice of L is a probability. *A state describes a mode of preparation (of a system for instance)*, (Jauch, 1968). *Then it becomes obvious that the state corresponds, roughly speaking, to the frequency of obtaining the answer "yes" for any test in the experimental conditions corresponding to that mode of preparation.* In what follows we will assume that for any atom there exists one and only one state taking the value 1 on that atom and a state cannot take the value 1 on two different atoms. This statement is supposed to be true for all theories used below. A state taking the value 1 on an atom is said to be pure. Since it is obvious that any pure state is completely defined by an atom, we introduce the notation δ_u, $u \in \Omega(L)$, for the state with the property $\delta_u(u) = 1$.

It is well known that the observables, i.e. those objects, which correspond in a given theory to physical quantities, are in the case of the theory L morphisms defined on the Borel algebra of subsets of a metric space and taking values in L (Piron, 1976). The metric space in discussion represents the possible values of the physical quantity of a given observable and is usually a subset of nth Cartesian power $\mathbf{R}^n$ of the set $\mathbf{R}$ of real numbers. Since in what follows we are not interested in values of the observables, any observable will be defined as a Boolean algebra in L. This definition is supported by the fact that the images of the above-mentioned morphisms are Boolean algebras. Of course, it is not possible to reproduce the morphism using its image only, but in our case this will be not necessary. The result of the measurement of an observable B in a state p may be defined as the probability $p{:}B \to [0, 1]$ Obviously, if the morphism m defining B is known, then the mapping $p \circ m$ is a probability whose interpretation is obvious.

Finally we introduce the notion of time evolution. A time evolution or simply an evolution is a family $\mathbf{V} = \{V_t; t \in \mathbf{R}\}$ of automorphisms of L. In what follows we will write simply $\mathbf{V} = (V_t)$. Automorphism of L is any bijective mapping $f : L \to L$ having the properties $f(\vee_i a_i) = \vee_i f(a_i)$, $f(a^\perp) = f(a)^\perp$. An evolution (V_t) describes the change in time of states as follows: given a state δ_u considered at the moment $t = 0$, the state at the moment t will be $\delta_u \circ V_t$, where "$\circ$" denotes the composition of functions. Since V_t is an automorphism, it may be easily proved that $\delta_u \circ V_t$ is a pure state. A simple but important property of automorphisms, which will be used later, is $\delta_u \circ V = \delta_{V^{-1}(u)}$, where V is an automorphism and u an atom of L. Indeed, since $\delta_u \circ V$ is a pure state, there exists a unique atom v such that $\delta_u \circ V = \delta_v$. Then we have $\delta_u(V(v)) = \delta_v(v) = 1$. From the properties of pure states in the theory L we get immediately $u = V(v)$ or $v = V^{-1}(u)$ and the equality is proved.

The structure of the paper is the following. The second paragraph contains the definition and some general properties of Fock theories in L. The third paragraph, which is the core of the paper studies Fock theories with time evolutions. The last section is devoted to comments of the obtained results and of their connections with the Hilbert-space formalism.

2. FOCK THEORIES

Let us consider L an orthomodular lattice having all properties of physical theories mentioned in Introduction. It will be assumed that L is also complete (any family of its elements has a join) although this property is not always necessary. $\mathbf{N}$ and $\mathbf{N}^n$ will denote the set of natural numbers and its nth Cartesian power, respectively. The elements of $\mathbf{N}^n$, which are of the form $(k_1, \ldots, k_n)$, will be denoted occasionally by small Greek letters. Let us consider a set $S = \{s_1, \ldots, s_n\}$ of n species of particles. Any element $(k_1, \ldots, k_n) = \alpha$ is a possible composition in particles of the species $s_1, \ldots, s_n$ respectively. It is assumed that for any composition there exists a test in L, denoted by $c(k_1, \ldots, k_n)$ or $c(\alpha)$, which will be called counting test corresponding to that composition. *It is obvious that, from the physical point of view, a counting test is a "yes–no" experiment, that permits one to establish if in an arbitrarily given state there is the composition corresponding to that test*. For the set S we may define a Fock theory as follows.

Definition 2.1. A Fock theory for the set S is a set of counting tests $F(S) = \{c(\alpha); \alpha \in \mathbf{N}^n\}$ having the following properties:

(a) $\underset{\alpha}{\vee} c(\alpha) = 1$;

(b) the test $\mathbf{V}(0,\ldots 0)$, called the vacuum of the theory $F(S)$, is orthogonal to any other test of the theory.

We might ask also another property for Fock theories: $\alpha \neq \beta \Rightarrow c(\alpha) \wedge c(\beta) = 0$. *The interpretation of this property results from the following reasoning:* $c(\alpha) \wedge c(\beta) \neq 0 \Rightarrow \exists u \in \Omega(L), u \leq c(\alpha), u \leq c(\beta) \Rightarrow \delta_u(c(\alpha)) = \delta_u(c(\beta)) = 1,$ *which means that in a state we have with probability* 1 *two different compositions. Such a conclusion is not at all physically plausible*. We did not ask this property in Definition 1 since it will be not necessary to use it in all cases discussed bellow.

For any Fock theory $F(S)$ we introduce a set $F_s(S)$ of so-called admissible states of the theory. That means that we do not accept *a priori* any δ_u as a possible state for particles described by the theory $F(S)$. The set $F_c(S) = \{\delta_u; u \in$

$\Omega(L), u \leq c(\alpha), \alpha \in N^n\}$ will be assumed included in $F_s(S)$ if we do not have arguments for rejecting some of them.

Definition 2.2. A Fock theory is called particle theory if any admissible state is different from zero for only one of its elements. It is said to be orthogonal if its elements are mutually orthogonal.

We will prove now an important property of particle Fock theories.

Proposition 2.1. *Let $F(S)$ be a particle Fock theory. Then $F(S)$ is orthogonal and $F_s(S) = F_c(S)$.*

Proof: Let us take $u \in \Omega(L)$, $u \leq c(\alpha)$. We have $\delta_u(c(\alpha)) = 1$. Then, since $F(S)$ is a particle theory, for any $c(\beta) \neq c(\alpha)$, $\delta_u(c(\beta)) = 0$. It results in $u \leq c(\beta)^{\perp}$, since for any $a \in L$ there exists a unique atom $v \leq a$ such that $\delta_u(a) = \delta_u(v)$, (Ivanov, 1987). Now it is easy to prove that $c(\alpha) \perp c(\beta)$, $\forall \beta \neq \alpha$ and the first part of the proposition is proved. For proving the second part, let us consider that $\delta_u \notin F_c(S)$. Therefore, since the elements of the theory are mutually orthogonal, we may write $\delta_u(\vee_{\alpha} c(\alpha)) = \Sigma_{\alpha} \delta_u(c(\alpha)) = 1$. Since $\forall \alpha, u$ is not under $c(\alpha)$, $\delta_u(c(\alpha)) \neq 1, \forall \alpha$. We get $\exists \alpha \neq \beta$, $\delta_u(c(\alpha)) \neq 0$, $\delta_u(c(\beta)) \neq 0$ and the proposition is completely proved. □

Suppose now that $F(S)$ is not orthogonal. Then we have two possible situations. One of them corresponds to the case when the counting tests are mutually compatible, but there are pairs of tests, $c(\alpha)$, $c(\beta)$, such that $c(\alpha) \wedge c(\beta) \neq 0$. The other assumes existence of pairs of tests, which are not compatible. In both cases the following proposition is true.

Proposition 2.2. *If $F(S)$ is not orthogonal, then there exists a state, which does not vanish on at least two different counting tests.*

Proof: Suppose $c(\alpha)$, $c(\beta)$ are not orthogonal counting tests. If we assume that for any atom $u \leq c(\alpha)$, $\delta_u(c(\beta)) = 0$, then it results immediately $c(\alpha) \perp c(\beta)$, which is not possible.

It is clear that the nontrivial situation in the case of nonorthogonal theories is that in which $c(\alpha) \wedge c(\beta) = 0$ for all pairs of counting tests. *From physical point of view particle theories describe "sharply defined particles." Indeed,* Proposition 1 *expresses the fact that in any admissible state we find certainly only one composition in particles. On the other hand if the particles are described by a nonorthogonal Fock theory, then we may affirm that the particles in question are* *not sharply defined entities, at least in the framework of that theory. That is because we can find states in which different compositions in particles have nonzero probabilities.* □

We end this paragraph with an interesting remark concerning orthogonal theories. Let $F(S) = \{c(k_1, \ldots, k_n); (k_1, \ldots, k_n) \in N^n\}$ be an orthogonal theory. By using counting tests of $F(S)$, we can define for any species s_i a Boolean algebra, which is in fact an observable corresponding to experimental procedures for measuring the number of particles of the species s_i. It is $N_i = \{\widehat{\bigvee_{\alpha^i_k}} c(\alpha^i_k); k \in \mathbf{N}\}$, where $\alpha^i_k = (k_1, \ldots, k_{i-1}, k, k_{i+1}, \ldots, k_n)$. It is obvious that $N_1, \ldots, N_n$ are mutually compatible Boolean algebras. *From the physical point of view this means that in our theory the number of particles from each species may be determined separately. It is also clear that the composition of any state may be determined by measuring simultaneously the number of particles of all species. When the theory, which describes the species from S is not orthogonal it is not possible to define observables for the number of particles of different species. Nevertheless, it is justified to talk about tests, which measure the composition of states in particles, corresponding to measurement procedures whose results may be interpreted assuming that particles in discussion are present in the system. It is important to observe that the impossibility to define observables for the number of particles in theories, which are not orthogonal is in accord to the fact that in such theories the particles are not sharply defined.*

3. FOCK THEORIES AND TIME EVOLUTIONS

In this section we intend to discuss some problems concerning the possibility of time evolutions to predict change of particles described by different Fock theories. We will have in view only conservative evolutions, i.e. evolutions having a Hamiltonian. The notion of Hamiltonian in the context of theories will defined below.

Definition 3.3. Let $\mathbf{V} = (V_t)$ be an evolution in the theory $\mathbf{L}$. We say that $a \in L$ is invariant under $\mathbf{V}$ and write $\mathbf{V}(a) = a$ if $V_t(a) = a$ for all $t \in \mathbf{R}$. If $G \subseteq L$ is a subset of L, we write (a, G)C if (a, g)C for all $g \in G$.

It is easy to verify that the set of all invariant under $\mathbf{V}$ elements of L, which will be denoted by L_v is an orthomodular sublattice of L. To prove this it is sufficient to apply the properties of automorphisms.

Definition 3.4. It is said that the evolution $\mathbf{V}$ has a Hamiltonian if there exists a Boolean algebra $H \subseteq L$ having the property (EH):$\mathbf{V}(a) = a \Leftrightarrow (a, H)$C. Such an evolution is called conservative if it is also a one-parameter group.

It is obvious that the set of all Boolean algebras having the property (EH) may be empty or may have more than one element. The following simple proposition gives us the possibility to define a Hamiltonian for a given evolution when the just mentioned set is not empty.

Proposition 3.3. *If the evolution* $\mathbf{V}$ *has at least one Boolean algebra with the property (EH), then there exists a greatest Boolean algebra with this property. It will be called the Hamiltonian of* $\mathbf{V}$.

Proof: Let H and H' be two Boolean algebras having the property (EH) with respect to the evolution $\mathbf{V}$. If $a \in H$, then $\mathbf{V}(a) = a$, which means (a, H')C. It results that the elements of the union of all Boolean algebras with the property (EH) are mutually compatible. Consequently, the orthomodular sublattice generated by this union is a Boolean algebra, which will be denoted by H_v. It remains to show that H_v has the property (EH). But this fact results immediately since all elements of H_v are joins and meets of elements of its generating set (Sikorski, 1964). *The Boolean algebra* H_v *is called Hamiltonian since it is strictly related with a conservative evolution, offering a criterion for finding invariant observables from the set of all possible observables of the theory. It is obvious that invariant observables must correspond to the so-called constants of motion.* □

It is obvious that for any $\mathbf{V}$, evolution with Hamiltonian, we have $H_v \subseteq L_v$. In general the following statement is true.

Proposition 3.4. *Let* $\mathbf{V}$ *be an evolution with Hamiltonian. Then* $L_v = H_v$ *if and only if* L_v *is a Boolean algebra.*

Proof: Take $a \in L_v$. Then we have (a, H_v)C and let us denote by B_a the Boolean algebra generated by the set $\{a\} \cap H_v$. We want to prove that B_a has the property (EH). If an element b is compatible with B_a, then it is also compatible with H_v, so that $b \in L_v$. Conversely, suppose that $b \in L_v$. Then, since L_v is a Boolean algebra, (a, b)C. We have also (b, H_v)C, so that (b, B_a)C is true. Since the Hamiltonian is the greatest Boolean algebra with the (EH) property, b is en element of H_v and the proposition is completely proved. □

It is important to observe that, if the lattice of all invariant under V is a Boolean algebra, it does not result that it has Hamiltonian. That is because from (a, L_v)C it does not result necessarily that a is invariant under V. In general, it is very difficult, if not impossible, to establish in this quite general framework criterions for deciding if an evolution has or not Hamiltonian. Consequently, in the sequel we will restrict ourselves to those physical theories, which have the property that *any conservative evolution is continuous* (see Comments). As is usually done, an evolution $\mathbf{V} = (V_t)$ is said to be continuous if for any state p and for any test $a \in L$ the function $t \mapsto p(V_t(a))$ is continuous.

We begin now to study time evolutions of Fock theories.

Definition 3.5.

(a) The evolution $\mathbf{V}$ is said to be admissible for the Fock theory $F(S)$ if $p \in F_s(S) \to p \circ V_t \in F_s(S)$ for all $t \in \mathbf{R}$.
(b) If $c(\alpha)$, $\alpha \in \mathbf{N}^n$ denote the counting tests of the theory $F(S)$, then for any species s_i and any state p we define a composition function as follows: $C_{ip}(t) = \Sigma_a k_{i\alpha} p(V_t c(\alpha))$.
(c) The evolution $\mathbf{V}$ is called reactive if at least one of its composition functions is not constant.

We will prove now a very important result concerning particle Fock theories.

Proposition 3.5. *Particle Fock theories have not admissible conservative reactive evolutions.*

Proof: Let $F(S)$ be a Fock theory whose counting tests and composition functions are denoted as in Definition 5(b). Let us consider an admissible pure state δ_u, $u \in \Omega(L)$ and the corresponding composition function $C_{i,\delta_u}(t) = \Sigma_\alpha k_{i\alpha}\delta_u$ $(V_t(c(\alpha)))$ defined with respect to the evolution $\mathbf{V}$. Suppose that the function $C_{i,\delta_u}(t) = C(t)$ is not constant. Then there are two alternatives:

(1) there exists $\tau > 0$ such that for any $t' > \tau$ we can find $t'' \in (\tau, t')$ with the property $(C(t'') \neq C(\tau)$;
(2) for any $\tau \geq 0$ there exists $t' > \tau$ such that $C(t'') = C(\tau)$ for all $t'' \in (\tau, t')$.

We will show that in both (1) and (2) cases $\mathbf{V}$ is not continuous. Since (2) is the negation of (1) the proposition will be proved. We prove first that if (1) is true, then $\mathbf{V}$ is not continuous. We show first that $\forall t' > \tau, \exists\, t_k \in (\tau, t')\delta_{V_{\tau(u)}}(V_{t_k}(u)) = 0$. Indeed, if this is not true, we would have for all $t \in (\tau, t')$, $V_\tau(u)$, $V_t(u) \leq c(\alpha)$. That is because the pure states corresponding to the atoms $V_\tau(u)$, $V_t(u)$ are admissible in a particle theory, so that if they are not orthogonal, then they are under the same counting test. But in this case we can write the following sequence of equalities: $\delta_u(V_t c(\alpha)) = \delta_{V_t^{-1}(u)}(c(\alpha)) = \delta_{V_\tau^{-1}(u)}(c(\alpha)) = \delta_u(V_\tau c(\alpha))$. This means that the composition function $C_{i,\delta_u}(t)$ is constant on the interval $[\tau, t')$, which contradicts the hypothesis. It results that we may find a sequence of real numbers $t_k \to \tau$ such that for all k $\delta_{V_{t_k}(u)}(V_\tau(u)) = 0 = \delta_u(V_{t_k}^{-1}(V_\tau(u)))$ and the limit of this sequence is obviously zero. On the other hand, if the evolution is supposed to be continuous, then we have obviously $\lim_{t_k \to \tau} \delta_u(V_{t_k}^{-1}(V_\tau(u))) = \delta_u(V_\tau^{-1}(V_\tau(u))) = \delta_u(u) = 1$, contradiction. It results that the evolution is not continuous. Suppose now that (2) is true. Then we can observe that there exists a denumerable family of intervals $(0, t_1), [t_1, t_2), \ldots, [t_{n-1}, t_n), \ldots$ whose union is $[0, \infty)$ and such that the composition function in question is constant on each of them. Since the composition function is not constant, there exist $t_{n-1}, t_n, C_{t,\delta_u}(t_{n-1}) \neq C_{i,\delta_u}(t_n)$. Then we take a sequence $t_k \to t_n$in$[t_{n-1}, t_n)$ and repeat practically the proof from the point (1). The proposition is completely proved. □

Proposition 5 has a simple but important consequence.

Proposition 3.6. *Any continuous admissible for a particle Fock theory evolution leaves invariant all counting tests of that theory.*

Proof: In Proposition 5 it has been proved in fact that, if there exists composition function, which is not constant, then the evolution is not continuous. This means that, if the admissible evolution $\mathbf{V}$ is continuous, then all composition functions are constant. Let us consider an atom $u \leq c(\alpha)$ and the composition functions associated to the pure state defined by u $C_{i,\delta_u}(t) = \Sigma_\alpha k_{i\alpha}\delta_{V_{t(u)}}(c(\alpha))$. Observe that the function $t \mapsto \delta_{V_{t(u)}}(c(\alpha))$ has as only possible values 0 and 1. Consequently, we may write $C_{i,\delta_u}(0) = k_{i\alpha}$. If $V_t(u)$ is not under $c(\alpha)$, then $C_{i,\delta_u}(t) = 0$. Further, if u is not under vacuum, then we may find $1 \leq i \leq n$, $k_{i\alpha} \neq 0$, so that the corresponding composition function is not constant. It results that all counting tests, which differ from vacuum, are invariant under $\mathbf{V}$. But, in this case the vacuum itself is invariant. □

Before going further, it is useful to point out that to any given Fock theory we may attach several admissible evolutions. That is because our definition of admissibility is a weak enough condition. For our purpose this definition is sufficient since our discussion is quite general and it is obvious that any derivation from some other conditions imposed by specific physical circumstances evolution must transform the set of admissible states into itself. Remember also that in what follows only conservative evolutions, i.e. evolutions with Hamiltonian will be considered.

Let us consider again $\mathbf{V}$ an evolution in the theory L and its Hamiltonian, which will be denoted for the moment by H. If $a \neq 0$ is an element of L, we introduce the following sets, which will be used below. They are $L_a = \{b \in L; b \leq a\}$, $H_a = \{a \wedge h; h \in H\}$ and $\mathbf{V}_a$, which is the restriction of all elements of $\mathbf{V}$ to L_a. We will prove the following proposition.

Proposition 3.7. *Given an evolution* $\mathbf{V}$ *in* L *and* H *its Hamiltonian, the following two assertions are true.*

(a) *L_a is an orthomodular lattice with the order inherited from L.*
(b) *H_a is the Hamiltonian of $\mathbf{V}_a$ in L_a if and only if $(a, H)C$.*

Proof: (a) The smallest and the greatest elements of L_a are respectively 0 and a. It is obvious that $b, d \in L_a \Rightarrow b \wedge d, b \vee d \in L_a$. Further, if $b \in L_a$, then from the orthomodularity of L we have $a = b \vee (a \wedge b^{\perp})$ and it becomes clear that the orthocomplement of b in L_a is $b^{\perp_a} = a \wedge b^{\perp}$. It is then easy to verify that "$\perp_a$" is orthocomplementation on L_a and L_a is orthomodular. For proving (b), let us take $b \in L_a$, (b, H_a)C. If $h \in H$, then from $h = (a \wedge h) \vee (a^{\perp} \wedge h)$ we get immediately (b, h)C, which implies $\mathbf{V}_a(b) = \mathbf{V}(b) = b$. If $\mathbf{V}(b) = b$, then (b, H)C and since (b, a)C, we get $(b, a \wedge h)$C for all $h \in H$. It results that H_a has the (EH) property, which confirms that it is a Hamiltonian. Conversely, suppose H_a is a Hamiltonian. Then $\mathbf{V}(a \wedge h) = a \wedge h$ for all $h \in H$. Since we have $a = \bigvee_{h \in H} (h \wedge a)$, one obtains immediately $\mathbf{V}(a) = a$ and finally (a, H)C. The proposition is completely proved. □

The Hamiltonian H_a may be called the reduction to a of the Hamiltonian H. Let us consider now $F(S)$ a Fock theory and $\mathbf{V}$ one of its admissible evolutions. If all counting tests $c(\alpha)$ are invariant under $\mathbf{V}$, then for any α there exists the reduction H_α of the Hamiltonian of $\mathbf{V}$ to $c(\alpha)$. If $F(S)$ is a particle theory, then it is clear that the pair $(F(S),\mathbf{V})$ may be replaced by the set $\{(L_\alpha, H_\alpha); \alpha \in \mathbf{N}^n\}$, where $L_\alpha = L_{c(\alpha)}$. Each of the pairs of this set represents a theory for a fixed number of particles from each species with the evolution corresponding to the Hamiltonian H_α. *A pair like $(F(S),\mathbf{V})$, i.e. having its properties will be called many particle theory. It is not difficult to understand that a many particle theory describes systems of particles, which are stable in some specific conservative conditions usually considered in concrete form of Hamiltonians. Moreover, one of the components of a many particle theory being a particle Fock theory, we know that there does not exist a conservative evolution able to consider change in time of the composition of systems. It is also important to remember that particle Fock theories describe sharply defined particles. On the other hand we know from experimental facts that there exist systems constituted from particles—like atoms and molecules—whose composition may change in conservative conditions. For describing processes in conservative conditions occurring in such systems, we need Fock theories, which might be orthogonal theories with an extended set of admissible states if compared with particle theories or theories, which are not orthogonal. The main appearing problem is how the good from physical point of view theories of these categories may be recognized or eventually constructed. Probably the most natural hypothesis is to assume that every system of particles is constituted in fact from stable in those conditions particles, which may produce composed nonsharply defined particles. The evolution of systems with composed particles inside will be generated by the evolution of the considered* *stable particles. We end the paragraph with the mathematical transcription of these considerations.*

Let us consider the Fock theories $F(S)$ and $(\phi(E), \mathbf{V}_\phi)$ in the physical theories L respectively Λ, where $(\phi(E), \mathbf{V}_\phi)$ is a many particle theory. Given $\boldsymbol{V}$ an evolution in L, we say that $(F(S),\boldsymbol{V})$ derives from $(\phi(E), \mathbf{V}_\phi)$ if there exists $f : L \to \Lambda$ an injective morphism of orthomodular lattices such that $f(\mathbf{V}(a)) = \mathbf{V}_\phi(f(a))$. *This definition has a quite clear meaning: E represnts the set of species of stable particles from which the particles of the species belonging to S are composed. The evolution* $\mathbf{V}_\phi$ *of stable particles is that, which generates the evolution of composed particles. Usually the evolutions for stable particles may be found taking into account basic physical considerations and some empirical facts, so that we get a good criterion for selecting, at least in principle, the evolutions for particles, which are not sharply defined.*

4. COMMENTS

The principal aim of the present work is to discuss the so-called Fock theories, which are methematical structures we introduced for studying systems containing particles whose number varies in time. These particles may be structureless or composed from other particles. All problems are discussed in the framework of quantum logics, called in our paper physical theories. The mathematical structure of quantum logic has been chosen since, in our opinion, it has one of the best physical interpretations if compared with other mathematical structures also called physical theories.

Two main conclusions emerge from our work. Both of them refer to the opportunity to consider the general mathematical structure of Fock space. The first of them is directly related to the possibility offered by Fock spaces to give a precise mathematical possibility for distinguishing between sharply defined particles and other particles whose existence is imposed by experimental facts, but they do not behave always as sharply defined entities. Such a distinction becomes possible in quite simple terms because Fock theories are thought to describe, each of them, all systems constituted from different number of particles of an arbitrarily given set of species. It has been seen in Paragraph 2 that distinction between the two mentioned types of particles may be done only in terms of compositions in particles of admissible states. The second conclusion is connected with conservative time evolutions of Fock theories. The results obtained from a careful enough analysis of such evolutions are quite interesting. First of all we find out that sharply defined particles do not change their number in conservative conditions. On the other hand, the other mentioned type of particles may change their number in conservative conditions and the most probable cause of this phenomenon is that they are in fact composed from other particles.

Finally some technical comments related with the evolutions as mathematical objects in our work. The fact that they appear as sets of automorphisms results since their elements must be symmetries of physical theories, (Ivanov, 1995). We had to show how to prescribe to any conservative evolution a Hamiltonian defined strictly in the mathematical framework of quantum logics. The definition, which has been proposed in Paragraph 3 is inspired by two fundamental results from Hilbert-space theory. They are spectral theorem for self-adjoint and unitary operators and Stone's theorem concerning unitary evolutions, (Stone, 1932). We have to point out that we used the continuity of evolutions only for proving the important Proposition 5. Unfortunately for the continuity of evolutions we could not find a satisfactory physical interpretation in terms of quantum logic system of notions. We only considered the fact that even in the Hilbert space formalism a sort of continuity for conservative evolutions must be required. On the other hand our "purely algebraic" definition permits to obtain a series of results, which confirm that the lattice-theoretical Hamiltonian has some characteristic properties of the usual Hilbert-space Hamiltonians. In this paper we derived and used only few lattice-theoretical results, those, which were necessary for our purpose. It seems that a more extended study concerning conservative evolutions and their Hamiltonians in this framework could be quite interesting.

REFERENCES

Ivanov, Al. (1987). *Revue Roumaine de Chimie* **32**, 1097.
Ivanov, Al. (1995). *International Journal of Theoretical Physics* **34**, 1487.
Jauch, J. M. (1968). *Foundations of Quantum Mechanics*, Addison-Wesley, Reading, MA.
Piron, C. (1976). *Foundations of Quantum Physics*, W. A. Benjamin, Inc.
Sikorski, R. (1964). *Boolean Algebras*, Springer-Verlag, New York.
Stone, M. H. (1932). *Annals of Mathematics* **33**(2), 643.